

An integral inequality for the invariant measure of a stochastic reaction–diffusion equation

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Dedicated to Jan Prüss

Abstract

We consider a reaction–diffusion equation perturbed by noise (not necessarily white). We prove an integral inequality for the invariant measure ν of a stochastic reaction–diffusion equation. Then we discuss some consequences as an integration by parts formula which extends to ν a basic identity of the Malliavin Calculus. Finally, we prove the existence of a surface measure for a ball and a half-space of H .

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1 Introduction and preliminaries

In the recent paper [DaDe14] the following inequality involving the invariant measure ν of the Burgers equation was proved,

$$\left| \int_H \langle (-A)^{-\alpha/2} D\varphi, z \rangle d\nu \right| \leq C_p \|\varphi\|_{L^p(H, \nu)} |z|, \quad \alpha > 1, \quad (1.1)$$

for all $\varphi \in C_b^1(H)$, $z \in H$ and all $p > 1$. Here A is the Laplace operator equipped with Dirichlet boundary conditions, $\alpha > 1$ and D represents the gradient. As noticed in that paper inequality (4.7) implies that $(-A)^{-\alpha/2} D$ is closable in $L^p(H, \nu)$ for all $p \geq 1$. Moreover, for each $z \in H$ there exists $v_z \in L^p(H, \nu)$ for all $p > 1$ such that

$$\int_H \langle (-A)^{-\alpha/2} D\varphi, z \rangle d\nu = \int_H v_z \varphi d\nu, \quad \forall \varphi \in C_b^1(H). \quad (1.2)$$

We recall that if $\nu = N_Q$ (the Gaussian measure of mean 0 and covariance Q), identity (1.2) with $A = -\frac{1}{2}Q^{-1}$ and $\alpha = 1$ is well known in Malliavin Calculus. In this case the

adjoint $(Q^{1/2}D)^*$ of $Q^{1/2}D$ is called the Skorhood operator. Moreover, (1.2) implies that ν is Fomin differentiable in all directions of $D((-A)^{\alpha/2})$ for $\alpha > 1$. For the definition of Fomin differentiability see e. g. [Pu98, Definition 1].

The aim of the present paper is to extend inequality (4.7) to the following reaction–diffusion equation in $H = L^2(\mathcal{O})$ where \mathcal{O} is a bounded domain of \mathbb{R}^n with sufficiently regular boundary.

$$\begin{cases} dX(t) = [AX(t) + p(X(t))]dt + (-A)^{-\gamma/2}dW(t), \\ X(0) = x. \end{cases} \quad (1.3)$$

where A is the realization of the Laplace operator Δ_ξ equipped with Dirichlet boundary conditions,

$$Ax = \Delta_\xi x, \quad x \in D(A), \quad D(A) = H^2(\mathcal{O}) \cap H_0^1(\mathcal{O}),$$

and $\frac{n}{2} - 1 < \gamma < 1$ (which obviously implies that $n \leq 3$). Moreover, p is a decreasing polynomial of odd degree equal to $N > 1$ and W is an H -valued cylindrical Wiener process on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t>0}, \mathbb{P})$.

It is well known that equation (1.3) has a unique strong solution and that the associate transition semigroup

$$P_t \varphi(x) = \mathbb{E}[\varphi(X(t, x))], \quad \varphi \in B_b(H)$$

possesses a unique invariant measure ν such that

$$M_m := \int_H |x|^m \nu(dx) < \infty, \quad \forall m \in \mathbb{N}, \quad (1.4)$$

see e.g. [Da04].

If $\gamma < \frac{n}{2} - 1$, equation (1.3) is not expected to have solutions with positive spatial regularity and the equation has to be renormalized. This has been studied in [DaDe03] for $n = 2$ and more recently in [Hai14] and [CaCh14] for $n = 3$.

Remark 1.1. All following results remain true replacing $(-A)^{-\gamma/2}$ with $B = G(-A)^{-\gamma/2}$ and $G \in L(H)$. Also, the assumption p decreasing could be replaced by p' bounded above.

The main result of the paper is the following

Theorem 1.2. *Let $\delta \in (0, 1 - \gamma)$, $p \in (1, \infty)$. Then there exists $C_p > 0$ such that for all $\varphi \in L^p(H, \nu)$ we have*

$$\int_H \langle D\varphi(x), h \rangle \nu(dx) \leq C_p \|\varphi\|_{L^p(H, \nu)} |h|_{1+\delta+\gamma}, \quad \forall h \in H^{1+\delta+\gamma}(\mathcal{O}).$$

Remark 1.3. Theorem 1.2 has already been proved in the paper [DaDe15] in the particular case when $\delta = 1 - \gamma$. As we shall see, the general case requires new tools as the estimates of $DX(t, x)$ in some Sobolev spaces, to which is devoted Section 2 below.

Let us describe the content of the paper. In Section 3 we use the estimates obtained in Section 2 to prove Theorem 1.2 and we discuss some consequences proving moreover a basic integration by parts formula, see (3.16) below. Finally, Section 4 is devoted to discuss surface integrals with respect to the measure ν , in particular for balls and half-spaces.

We conclude this section with some notation and preliminary. The norm of $H = L^2(\mathcal{O})$ will be denoted by $|\cdot|$ and the inner product by $\langle \cdot, \cdot \rangle$. For $p \geq 1$, $|\cdot|_{L^p}$ is the norm of $L^p(\mathcal{O})$. The operator A is self-adjoint negative and there exist an orthonormal basis (e_h) in H and an increasing sequence of positive numbers (α_h) such that

$$Ae_h = -\alpha_h e_h, \quad \forall h \in \mathbb{N}. \quad (1.5)$$

For any $\alpha \in \mathbb{R}$, $(-A)^\alpha$ denotes the α power of the operator $-A$ and $|\cdot|_\alpha$ is the norm of $D((-A)^{\alpha/2})$ which is equivalent to the norm of the Sobolev space $H^\alpha(\mathcal{O})$. We have $|\cdot|_0 = |\cdot| = |\cdot|_{L^2}$. We shall use the interpolatory estimate

$$|x|_b \leq |x|_a^{\frac{c-b}{c-a}} |x|_c^{\frac{b-a}{c-a}}, \quad -\infty < a < b < c < +\infty. \quad (1.6)$$

Let $\alpha \in (0, 1)$. By the Sobolev embedding theorem we have $H^\alpha(\mathcal{O}) \subset L^q(\mathcal{O})$, where $q = \frac{2n}{n-\alpha}$. By duality it follows that $L^{\frac{2n}{n+\alpha}}(\mathcal{O}) \subset H^{-\alpha}(\mathcal{O})$ and therefore

$$|x|_{-\alpha} \leq c |x|_{\frac{2n}{n+\alpha}}, \quad \forall \alpha \in (0, 1), x \in L^{\frac{2n}{n+\alpha}}(\mathcal{O}), \quad (1.7)$$

for a suitable constant $c > 0$.

Moreover, it is convenient to introduce the following approximating problem

$$\begin{cases} dX_\epsilon(t) = (AX_\epsilon(t) + p_\epsilon(X_\epsilon(t))dt + (-A)^{-\gamma/2}dW(t), \\ X_\epsilon(0) = x \in H, \end{cases} \quad (1.8)$$

where for any $\epsilon > 0$, p_ϵ are the Yosida approximations of p , that is

$$p_\epsilon(r) = \frac{1}{\epsilon} (r - J_\epsilon(r)), \quad J_\epsilon(r) = (1 - \epsilon p(\cdot))^{-1}(r), \quad r \in \mathbb{R}.$$

Let us denote by P_t^ϵ the approximate transition semigroup

$$P_t^\epsilon \varphi(x) = \mathbb{E}[\varphi(X_\epsilon(t, x))], \quad \varphi \in B_b(H)$$

and recall the following Bismut-Elworthy-Li formula, see [Ce01].

$$\langle DP_t^\epsilon \varphi(x), h \rangle = \frac{1}{t} \mathbb{E} \left[\varphi(X_\epsilon(t, x)) \int_0^t \langle (-A)^{\frac{\gamma}{2}} \eta_\epsilon^h(s, x), dW(s) \rangle \right], \quad h \in H, \quad (1.9)$$

where $\eta_\epsilon^h(t, x) =: DX_\epsilon(t, x) \cdot h$ is the x -derivative of $X_\epsilon(t, x)$ in the direction h . As well known, $\eta_\epsilon^h(t, x)$ is the solution of the equation

$$\begin{cases} \frac{d}{dt} \eta_\epsilon^h(t, x) &= A \eta_\epsilon^h(t, x) - p'_\epsilon(X_\epsilon(t, x)) \eta_\epsilon^h(t, x), \\ \eta_\epsilon^h(0, x) &= h. \end{cases} \quad (1.10)$$

Let us recall the following elementary but useful identity (proved in [DaDe15]). For any $\varphi \in C_b^1(H)$, $\epsilon > 0$, $h, x \in D(A)$, we have

$$P_t^\epsilon(\langle D\varphi, h \rangle) = \langle DP_t^\epsilon \varphi, h \rangle - \int_0^t P_{t-s}^\epsilon(\langle Ah + p'_\epsilon h, DP_s^\epsilon \varphi \rangle) ds. \quad (1.11)$$

2 Estimates of $\eta_\epsilon^h(t, x)$ and of $DP_t^\epsilon \varphi(x)$

Let us fix $T > 0$, $x \in H$. We are going to state some estimates of $\eta_\epsilon^h(t, x)$ in the norms $|\cdot|_{-\alpha}$ and $|\cdot|_{1-\alpha}$, needed below. We shall denote by $\Delta_{T,\epsilon}(X)$ the random variable

$$\Delta_{T,\epsilon}(X) = \sup_{t \in [0, T]} |p'_\epsilon(X_\epsilon(t, x))|_\infty^2 + 1. \quad (2.1)$$

Lemma 2.1. *For any $\alpha \in (0, 1)$ we have*

$$\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha}^2 dt \leq C(T) \Delta_{T,\epsilon}(X) |h|_{-\alpha}^2, \quad \forall h \in H^{-\alpha}(\mathcal{O}). \quad (2.2)$$

Proof. We shall proceed by duality. Let us consider the mapping

$$U : H \rightarrow L^2(0, T; H), \quad h \mapsto \eta^h(\cdot, x),$$

(here $x \in H$ and $\omega \in \Omega$ are fixed.) Let us find the adjoint U^* of U . Write for $h \in H$, $F \in L^2(0, T; H)$,

$$\begin{aligned} \langle U(h), F \rangle_{L^2(0, T; H)} &= \int_0^T \langle \eta_\epsilon^h(t, x), F(t) \rangle dt \\ &= \int_0^T \langle G(t, 0, x)h, F(t) \rangle dt = \left\langle h, \int_0^T G(t, 0, x)F(t) dt \right\rangle, \end{aligned}$$

where $G(t, s, x)$ is the evolution operator of the family of linear operators $(A + p'_\epsilon(X(t, x)))_{t \in [0, T]}$. Noticing the $G(t, s, x)$ is symmetric we have

$$U^*(F) = \zeta(0), \quad \forall F \in L^2(0, T; H), \quad (2.3)$$

where ζ is the solution of the problem

$$\begin{cases} \zeta'(t) = -A\zeta(t) - p'_\epsilon(X(t, x))\zeta(t) + F(t), \\ \zeta(T) = 0. \end{cases} \quad (2.4)$$

Now (2.2) is equivalent to

$$\int_0^T |(-A)^{\frac{1-\alpha}{2}} U(h)(s)|^2 ds \leq C(T) \Delta_{T,\epsilon}(X) |(-A)^{-\frac{\alpha}{2}} h|^2, \quad (2.5)$$

which by duality it equivalent to

$$|(-A)^{\frac{\alpha}{2}} \zeta(0)|^2 \leq C(T) \Delta_{T,\epsilon}(X) \int_0^T |(-A)^{\frac{\alpha-1}{2}} F(t)|^2 dt. \quad (2.6)$$

To prove (2.6), we shall proceed in two steps.

Step 1. We have

$$|\zeta(t)|^2 \leq \int_0^T |F(s)|_{-1}^2 ds \quad (2.7)$$

Taking the inner product in (2.4) with ζ and recalling that $p'_\epsilon(r) \leq 0$, we find

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} |\zeta(t)|^2 &= |\zeta(t)|_1^2 - \int_{\mathcal{O}} p'_\epsilon(X_\epsilon(t, x)) \zeta(t)^2 dx + \int_{\mathcal{O}} F(t) \zeta(t) dx \\ &\geq |\zeta(t)|_1^2 - |\zeta(t)|_1 |F(t)|_{-1} \\ &\geq \frac{1}{2} |\zeta(t)|_1^2 - \frac{1}{2} |F(t)|_{-1}^2. \end{aligned}$$

It follows that

$$\frac{d}{dt} |\zeta(t)|^2 \geq |F(t)|_{-1}^2, \quad \forall t \in [0, T].$$

Integrating from t and T , yields (2.7).

Step 2. Conclusion.

Now we take the inner product in (2.4) with $(-A)^\alpha \zeta$,

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} |\zeta(t)|_\alpha^2 &= |\zeta(t)|_{\alpha+1}^2 - \int_{\mathcal{O}} p'_\epsilon(X_\epsilon(t, x)) \zeta(t) (-A)^\alpha \zeta(t) dx \\ &\quad + \int_{\mathcal{O}} F(t) (-A)^\alpha \zeta(t) dx \\ &\geq |\zeta(t)|_{\alpha+1}^2 - |p'_\epsilon(X_\epsilon(t, x))|_\infty |\zeta(t)| |\zeta(t)|_{2\alpha} - |\zeta(t)|_{\alpha+1} |F(t)|_{\alpha-1}. \end{aligned}$$

Since $\alpha \leq 1$ by the Poincaré inequality we have

$$|\zeta(t)|_{2\alpha} \leq c |\zeta(t)|_{\alpha+1}$$

and consequently

$$\frac{1}{2} \frac{d}{dt} |\zeta(t)|_\alpha^2 \geq -c |p'_\epsilon(X(t, x))|_\infty^2 |\zeta(t)|^2 - |F(t)|_{\alpha-1}^2.$$

Integrating from 0 and T , yields

$$|\zeta(0)|_\alpha^2 \leq c |p'_\epsilon(X_\epsilon(t, x))|_\infty^2 \int_0^T |\zeta(t)|^2 dt + \int_0^T |F(t)|_{\alpha-1}^2 dt. \quad (2.8)$$

Now by (2.7) we deduce

$$|\zeta(t)|^2 \leq \int_t^T |F(s)|_{-1}^2 ds \leq C(T) \int_0^T |F(s)|_{\alpha-1}^2 ds. \quad (2.9)$$

Substituting in (2.8), yields

$$|\zeta(0)|_\alpha^2 \leq C(T) \Delta_{T,\epsilon}(X) \int_0^T |F(s)|_{\alpha-1}^2 ds,$$

as required. \square

Lemma 2.2.

$$|\eta_\epsilon^h(t, x)|_{-\alpha} \leq C(T) \Delta_{T,\epsilon}(X) |h|_{-\alpha}^2, \quad \forall h \in H^{-\alpha}(\mathcal{O}). \quad (2.10)$$

Proof. Fix $t \in [0, T]$ and introduce a mapping

$$V_t : H \rightarrow H, \quad h \rightarrow \eta_\epsilon^h(t, x),$$

(where $x \in H$ and $\omega \in \Omega$) are fixed. Then $V_t^*(h) = G(t, 0)k$ and so $V_t^*(k) = \zeta(0)$ where ζ is the solution to

$$\begin{cases} \zeta'(t) = -A\zeta(t) - p'_\epsilon(X(t, x))\zeta(t), \\ \zeta(t) = k. \end{cases} \quad (2.11)$$

We have clearly

$$|\zeta(s)|^2 \leq |k|^2, \quad \forall s \in [0, t]. \quad (2.12)$$

To prove (2.10), arguing by duality, it is enough to prove that

$$|\zeta(t)|_\alpha^2 \leq C(T) \Delta_{T,\epsilon}(X) |h|_\alpha^2. \quad (2.13)$$

Write

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} |\zeta(t)|_\alpha^2 &= |\zeta(t)|_{\alpha+1}^2 - \int_{\mathcal{O}} p'_\epsilon(X_\epsilon(t, x)) \zeta(t) (-A)^\alpha \zeta(t) dx \\ &\geq |\zeta(t)|_{\alpha+1}^2 - |p'(X_\epsilon(t, x))|_\infty |\zeta(t)| |\zeta(t)|_{-2\alpha} \\ &\geq -c |p'(X_\epsilon(t, x))|_\infty |\zeta(t)|^2. \end{aligned} \quad (2.14)$$

Therefore, thanks to (2.12),

$$\begin{aligned} |\zeta(t)|_\alpha^2 &\leq |k|_\alpha^2 + \int_0^t |p'_\epsilon(X_\epsilon(s, x))|_\infty^2 |\zeta(s)|^2 ds \\ &\leq |k|_\alpha^2 + \sup_{t \in [0, T]} |p'_\epsilon(X_\epsilon(s, x))|_\infty^2 t |k|^2 \\ &\leq C(T) \Delta_{T,\epsilon}(X) |k|_\alpha^2 \end{aligned}$$

as required. \square

Corollary 2.3. *Let $\delta \in (0, 1 - \alpha)$, then*

$$\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha-\delta}^2 dt \leq C(T) T^\delta \Delta_{T,\epsilon}(X) |h|_{-\alpha}^2. \quad (2.15)$$

Proof. By (1.6) with $a = -\alpha$, $b = 1 - \alpha - \delta$, $c = 1 - \alpha$, we have

$$\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha-\delta}^2 dt \leq \int_0^T |\eta_\epsilon^h(t, x)|_{-\alpha}^{2\delta} |\eta_\epsilon^h(t, x)|_{1-\alpha}^{2(1-\delta)} dt. \quad (2.16)$$

Now, taking into account (2.10), we have

$$\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha-\delta}^2 dt \leq C(T)^\delta (\Delta_{T,\epsilon}(X))^\delta |h|_{-\alpha}^{2\delta} \int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha}^{2(1-\delta)} dt. \quad (2.17)$$

By Hölder's inequality with exponents $\frac{1}{1-\delta}$, $\frac{1}{\delta}$, we deduce

$$\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha-\delta}^2 dt \leq C(T)^\delta (\Delta_{T,\epsilon}(X))^\delta |h|_{-\alpha}^{2\delta} T^\delta \left(\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha}^2 dt \right)^{1-\delta}. \quad (2.18)$$

Finally, taking into account (2.2), yields

$$\int_0^T |\eta_\epsilon^h(t, x)|_{1-\alpha-\delta}^2 dt \leq C(T) \left(\sup_{t \in [0, T]} |p'_\epsilon(X_\epsilon(t, x))|_\infty^2 + 1 \right) |h|_{-\alpha}^2, \quad (2.19)$$

and (2.15) follows. \square

Now we state an estimate for $DP_t^\epsilon \varphi(x)$. Fix $\delta \in (0, 1 - \gamma)$ and set $\alpha = 1 - \gamma - \delta$.

Lemma 2.4. *Let $p > 1$, $\delta \in (0, 1 - \gamma)$ and set $\alpha = 1 - \gamma - \delta$. Then we have*

$$|\langle DP_t^\epsilon \varphi(x), h \rangle| \leq C(t) t^{\frac{\delta}{2}-1} [\mathbb{E}(\varphi^p(X_\epsilon(t, x)))]^{1/p} \left[\mathbb{E} \left((\Delta_{T,\epsilon}(X))^{p'/2} \right) \right]^{1/p'} |h|_{-\alpha}, \quad (2.20)$$

with $\frac{1}{p} + \frac{1}{p'} = 1$.

Proof. By (1.9) and Hölder's inequality we have

$$|\langle DP_t^\epsilon \varphi(x), h \rangle| \leq \frac{1}{t} [\mathbb{E}(\varphi^p(X_\epsilon(t, x)))]^{1/p} \left[\mathbb{E} \left(\left(\int_0^t \langle (-A)^{\frac{\gamma}{2}} \eta_\epsilon^h(s, x), dW(s) \rangle \right)^{p'} \right) \right]^{1/p'}, \quad h \in H, \quad (2.21)$$

Thanks to Burkholder's inequality and recalling that $\gamma = 1 - \alpha - \delta$ it follows that

$$\begin{aligned} |\langle DP_t^\epsilon \varphi(x), h \rangle| &\leq \frac{3}{t} [\mathbb{E}(\varphi^p(X_\epsilon(t, x)))]^{1/p} \\ &\times \left\{ \mathbb{E} \left[\left(\int_0^t |\eta_\epsilon^h(s, x)|_{1-\alpha-\delta}^2 ds \right)^{p'/2} \right] \right\}^{1/p'}, \quad h \in H. \end{aligned} \quad (2.22)$$

Now (2.21) follows from Corollary 2.3. \square

3 Integral estimates of DP_t^ϵ

Lemma 3.1. *Let $p > 1$, $q > \frac{p}{p-1}$, $\delta \in (0, 1 - \gamma)$ and set $\alpha = 1 - \gamma - \delta$. Then there exists $C > 0$ such that*

$$\int_H |\langle DP_t^\epsilon \varphi(x), h(x) \rangle| \nu(dx) \leq C t^{\frac{\delta}{2}-1} \|\varphi\|_{L^p(H, \nu)} \|h\|_{- \alpha} \|h\|_{L^q(H, \nu)}, \quad \forall h \in L^q(H, \nu), \varphi \in C_b^1(H). \quad (3.1)$$

Proof. By (2.20) we deduce

$$|\langle DP_t^\epsilon \varphi(x), h(x) \rangle| \leq C(t) t^{\frac{\delta}{2}-1} [\mathbb{E}(\varphi^p(X_\epsilon(t, x)))]^{1/p} \left[\mathbb{E} \left(\Delta_{T, \epsilon}(X)^{p'/2} \right) \right]^{1/p'} |h(x)|_{- \alpha}. \quad (3.2)$$

Now we integrate (3.2) with respect to ν over H and use a triple Hölder's inequality with exponents p, q, r such that $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$.

$$\begin{aligned} \int_H |\langle DP_t^\epsilon \varphi(x), h(x) \rangle| \nu(dx) &\leq C(t) t^{\frac{\delta}{2}-1} \left(\int_H P_t(\varphi^p) d\nu \right)^{1/p} \\ &\times \left(\int_H \left\{ \mathbb{E} \left[(\Delta_{T, \epsilon}(X)^{p'/2}) \right] \right\}^{\frac{r}{p'}} d\nu \right)^{\frac{1}{r}} \left(\int_H |h(x)|_{- \alpha}^q d\nu \right)^{\frac{1}{q}}. \end{aligned} \quad (3.3)$$

But, recalling that p is a polynomial of degree N , we see that there exists $C_1 > 0$ such that

$$(\Delta_{T, \epsilon}(X)^{p'/2} \leq C_1 \sup_{t \in [0, T]} |X_\epsilon(t, x)|_\infty^{\frac{p'(N-1)}{2}} + C_1.$$

Now by [Da04, Theorem 4.8(iii)], there is $C_2 > 0$ such that

$$\mathbb{E} \left[(\Delta_{T, \epsilon}(X)^{p'/2}) \right] \leq C_2 |x|^{\frac{p'(N-1)}{2}} + C_2$$

and so there is $C_3 > 0$ such that

$$\left\{ \mathbb{E} \left[(\Delta_{T, \epsilon}(X)^{p'/2}) \right] \right\}^{\frac{r}{p'}} \leq C_3 |x|^{\frac{r(N-1)}{2}} + C_3. \quad (3.4)$$

Consequently, thanks to (1.4) we have

$$\int_H \left\{ \mathbb{E} \left[(\Delta_{T, \epsilon}(X)^{p'/2}) \right] \right\}^{\frac{r}{p'}} d\nu = C_4 < \infty. \quad (3.5)$$

Substituting (3.5) into (3.3) and taking into account the invariance of ν yields the conclusion. \square

We are now ready to show the main result of the paper.

Theorem 3.2. *Let $\delta \in (0, 1 - \gamma)$, $p \in (1, \infty)$. Then there exists $C_p > 0$ such that for all $\varphi \in C_b^1(H, \nu)$ we have*

$$\int_H \langle D\varphi(x), h \rangle \nu(dx) \leq C \|\varphi\|_{L^p(H, \nu)} |h|_{1+\delta+\gamma}, \quad \forall h \in H^{1+\delta+\gamma}(\mathcal{O}). \quad (3.6)$$

Proof. Let us integrate identity (1.11) with respect to ν_ϵ over H . Taking into account the invariance of ν_ϵ , we obtain

$$\begin{aligned} \int_H \langle D\varphi(x), h \rangle \nu_\epsilon(dx) &= \int_H \langle DP_t^\epsilon \varphi(x), h \rangle \nu_\epsilon(dx) \\ &\quad - \int_0^t \int_H \langle Ah + p'_\epsilon(x)h, DP_s^\epsilon \varphi(x) \rangle ds \nu_\epsilon(dx) \end{aligned} \quad (3.7)$$

$$=: J_1 + J_2.$$

As for J_1 we have by (3.1)

$$J_1 = \int_H \langle DP_t^\alpha \varphi(x), h \rangle \nu_\epsilon(dx) \leq Ct^{-1+\delta/2} \|\varphi\|_{L^p(H, \nu_\epsilon)} |h|_{-\alpha}. \quad (3.8)$$

As for J_2 we have again by (3.1), taking into account that $|Ah|_\alpha = |h|_{2-\alpha} = |h|_{1+\delta+\gamma}$,

$$\begin{aligned} J_2 &= \int_0^t \int_H \langle Ah + p'_\epsilon(x)h, DP_s^\epsilon \varphi(x) \rangle ds d\nu_\epsilon \\ &\leq C \int_0^t s^{-1+\delta/2} ds \|\varphi\|_{L^p(H, \nu_\epsilon)} (|h|_{1+\delta+\gamma} + \|p'_\epsilon h\|_{-\alpha})_{L^q(H, \nu_\epsilon)} \end{aligned} \quad (3.9)$$

On the other hand, thanks to (1.7) and Hölder's inequality, we have

$$|p'_\epsilon h|_{-\alpha} \leq c |p'_\epsilon h|_{L^{\frac{2n}{n+\alpha}}} \leq c |p'_\epsilon|_{L^{\frac{2n}{\alpha}}} |h|_{L^2}.$$

Moreover thanks to [Da04, Proposition 4.20], there is $c' > 0$ such that

$$\| |p'_\epsilon h|_{-\alpha} \|_{L^q(H, \nu_\epsilon)} \leq c \left(\int_H |p'_\epsilon|_{L^{\frac{2n}{\alpha}}}^q d\nu_\epsilon \right)^{1/q} |h| \leq c' |h|.$$

Consequently

$$J_2 \leq C \int_0^t s^{-1+\delta/2} ds \|\varphi\|_{L^p(H, \nu_\epsilon)} (|h|_{1+\delta+\gamma} + c' |h|). \quad (3.10)$$

By (3.8) and (3.10), setting $t = 1$ we find for a suitable constant C_p

$$\int_H \langle D\varphi(x), h \rangle \nu_\epsilon(dx) \leq C \|\varphi\|_{L^p(H, \nu_\epsilon)} |h|_{1+\delta+\gamma}, \quad \forall h \in H^{1+\delta+\gamma}(\mathcal{O}). \quad (3.11)$$

Letting $\epsilon \rightarrow 0$, yields (3.6) (see [DaDe15, Proposition 14]).

□

3.1 Some consequences of Theorem 3.2.

In the following we shall assume that the assumptions of Theorem 3.2 are fulfilled and set

$$\beta = \frac{1 + \gamma + \delta}{2}.$$

(Notice that $\beta > \frac{1}{2}$.) We shall write (3.6) as

$$\int_H \langle (-A)^{-\beta} D\varphi(x), h \rangle \nu(dx) \leq C \|\varphi\|_{L^p(H, \nu)} |h|, \quad \forall \varphi \in C_b^1(H), h \in H. \quad (3.12)$$

The proof of the following result is similar to [DaDe15, Proposition 11], so, it will be omitted.

Proposition 3.3. *The linear operator*

$$\varphi \in C_b^1(H) \subset L^p(H, \nu) \mapsto (-A)^{-\beta} D\varphi \in L^p(H, \nu; H),$$

is closable in $L^p(H, \nu)$ for all $p > 1$.

We shall denote by M_p the closure of $(-A)^{-\beta} D$ and by M_p^* the dual of M_p , so that we have

$$\int_H \langle M_p \varphi, F \rangle d\nu = \int_H \varphi M_p^*(F) d\nu, \quad \forall \varphi \in D(M_p), F \in D(M_p^*). \quad (3.13)$$

Clearly for any $p \in (0, 1)$,

$$M_p : D(M_p) \subset L^p(H, \nu) \rightarrow L^p(H, \nu; H)$$

and

$$M_p^* : D(M_p^*) \subset L^q(H, \nu; H) \rightarrow L^q(H, \nu),$$

where $q = \frac{p}{p-1}$.

When no confusion may arise we shall drop the sub-index p .

Remark 3.4. Obviously, $F \in D(M_p^*)$ iff there exists $K > 0$ such that

$$\left| \int_H \langle (-A)^{\beta} D\varphi, F \rangle d\nu \right| \leq K \|\varphi\|_{L^p(H, \nu)}, \quad \forall \varphi \in C_b^1(H). \quad (3.14)$$

In this case we have

$$\|M_p^*(F)\|_{L^q(H, \nu; H)} \leq K. \quad (3.15)$$

Let us show an integration by parts formula.

Proposition 3.5. *For any $z \in H$ there exists a function $v_z \in L^q(H, \nu)$ for all $q \in (1, \infty)$ such that*

$$\int_H \langle (-A)^{-\beta} D\varphi(x), z \rangle \nu(dx) = \int_H \varphi(x) v_z(x) \nu(dx) \quad (3.16)$$

and

$$\|v_z\|_{L^q(H, \nu)} \leq C_p |z|, \quad (3.17)$$

where $p = \frac{q}{q-1}$.

Therefore there exists the Fomin derivative of ν in all directions of $D((-A)^\beta)$.

Proof. Set $F_z(x) = h$ for all $x \in H$. By (3.12) we deduce

$$\left| \int_H \langle (-A)^{-\beta} D\varphi(x), F_z(x) \rangle \nu(dx) \right| \leq C \|\varphi\|_{L^p(H, \nu)} |z|, \quad \forall \varphi \in C_b^1(H), z \in H. \quad (3.18)$$

By Remark 3.4 it follows that $F_z \in D(M_p^*)$, so that setting $v_z = M_p^*(F_z)$ we see that (3.17) holds. \square

Now, recalling (1.5) we find

Corollary 3.6. *For all $h \in \mathbb{N}$ we have*

$$\int_H D_h \varphi(x) \nu(dx) = \alpha_h^\beta \int_H \varphi(x) v_h(x) \nu(dx) \quad (3.19)$$

where $v_h = v_{e_h}$ and

$$\|v_h\|_{L^q(H, \nu)} \leq C_p, \quad (3.20)$$

where $p = \frac{q}{q-1}$.

Let us now find an expression of M^* .

Proposition 3.7. *Let $F = \sum_{h=1}^m f_h e_h$ with $f_h \in C_b^1(H)$ and $m \in \mathbb{N}$. Then $F \in D(M^*)$ and it results*

$$M^*(F) = -\operatorname{div} [(-A)^{-\beta} F] + \sum_{h=1}^m f_h v_h. \quad (3.21)$$

Proof. Write

$$\begin{aligned} \int_H \langle (-A)^{-\beta} D\varphi, F \rangle d\nu &= \sum_{h=1}^m \alpha_h^{-\beta} \int_H D_h \varphi f_h d\nu \\ &= \sum_{h=1}^m \alpha_h^{-\beta} \int_H D_h(\varphi f_h) d\nu - \sum_{h=1}^m \alpha_h^{-\beta} \int_H \varphi D_h f_h d\nu \end{aligned}$$

Thanks to Corollary 3.6 we have

$$\int_H \langle (-A)^{-\beta} D\varphi, F \rangle d\nu = \sum_{h=1}^m \int_H \varphi v_h f_h d\nu - \int_H \varphi \operatorname{div} [(-A)^{-\beta} F] d\nu$$

which implies (3.21). \square

4 Application to surface integrals

We still assume in this section that the assumptions of Theorem 3.2 are fulfilled and set $\beta = \frac{1+\gamma+\delta}{2}$.

Let moreover $g : H \rightarrow \mathbb{R}$ be a Borel function. For all $\varphi \in L^1(H, \nu)$ we set

$$G_\varphi(r) = \int_{\{g \leq r\}} \varphi d\nu, \quad r \in g(H).$$

Definition 4.1. Assume that $G_\varphi(\cdot)$ is C^1 for any $\varphi \in C_b(H)$ and for $r \in g(H)$ there exists a measure σ_r concentrated on $\{g = r\}$ such that

$$D_r G_\varphi(r) = \lim_{\epsilon \rightarrow 0} \frac{1}{2\epsilon} \int_{\{r-\epsilon \leq g \leq r+\epsilon\}} \varphi d\nu = \int_{\{g=r\}} \varphi d\sigma_r.$$

Then σ_r is called the surface measure on $\{g = r\}$ determined by ν .

When ν is Gaussian several papers have been devoted to prove existence of surface measures, starting from the pioneering works by Airault and Malliavin, [AiMa88] and Fejer-de La Pradelle, [FePr92]; we also quote [DaLuTu14]. The typical assumption is

$$F_g := \frac{Mg}{|Mg|^2} \in D(M^*). \quad (4.1)$$

Concerning the measure ν , the following result has been proved in [DaLuTu] as a simple generalization of [DaLuTu14].

Theorem 4.2. Let M be the closure of $(-A)^{-\beta} D$. Assume that $g \in D(M)$ and

$$F_g := \frac{Mg}{|Mg|^2} \in D(M^*). \quad (4.2)$$

Then there exists a surface measure σ_r on $\{g = r\}$ determined by ν .

Remark 4.3. A similar result was obtained by [Pu98] and [BoMa], under the assumption that the Fomin derivative of ν exists in sufficiently many directions $z \in H$ where identity (3.19) is fulfilled.

Notice that by (3.21) it follows that (formally)

$$\begin{aligned} & M^* \left(\frac{(-A)^{-\beta} Dg}{|(-A)^{-\beta} Dg|^2} \right) \\ &= \operatorname{div} \left[\frac{(-A)^{-2\beta} Dg}{|(-A)^{-\beta} Dg|^2} \right] + \frac{1}{|(-A)^{-\beta} Dg|^2} \sum_{h=1}^{\infty} \alpha_h^{-\beta} v_h D_h g. \end{aligned} \quad (4.3)$$

In this section we shall prove that condition (4.2) is fulfilled for balls and half-spaces.

4.1 Surface measure of balls

In this subsection we shall assume $n = 1$ so that

$$\sum_{h=1}^{\infty} \alpha_h^\beta < \infty, \quad (4.4)$$

because $\beta > 1/2$. Let $g(x) = |x|^2$ and take $r > 0$, so that the surface $\{g = r\}$ coincides with the ball $B_{\sqrt{r}}$ in H with center 0 and radius \sqrt{r} . To apply Theorem 4.2 we have to show that $F_g \in D(M^*)$, where

$$F_g(x) = \frac{(-A)^{-\beta} x}{2|(-A)^{-\beta} x|^2}.$$

To this aim it is convenient to introduce a regular finite dimensional approximation of F_g setting

$$F_g^n(x) = \frac{(-A)^{-\beta} P_n x}{n^{-1} + 2|(-A)^{-\beta} x|^2},$$

where P_n is the orthogonal projector on $\text{span}(e_1, \dots, e_n)$. Clearly $F_g^n \in D(M^*)$ and by (3.21) we have

$$M^*(F_g^n)(x) = -\text{div} \left[\frac{(-A)^{-2\beta} P_n x}{n^{-1} + 2|(-A)^{-\beta} x|^2} \right] + \frac{\sum_{h=1}^n \alpha_h^{-\beta} x_h v_h}{n^{-1} + 2|(-A)^{-\beta} x|^2}. \quad (4.5)$$

But for $j = 1, \dots, n$ we have

$$D_j \left[\frac{(-A)^{-2\beta} x_j}{n^{-1} + 2|(-A)^{-\beta} x|^2} \right] = \frac{\alpha_j^{-2\beta}}{n^{-1} + 2|(-A)^{-\beta} x|^2} - \frac{4\alpha_j^{-4\beta} x_j^2}{(n^{-1} + 2|(-A)^{-\beta} x|^2)^2}.$$

Therefore

$$M^*(F_g^n)(x) = -\frac{\text{Tr}[P_n(-A)^{-2\beta}]}{n^{-1} + 2|(-A)^{-\beta} x|^2} + \frac{4|(-A)^{-2\beta} P_n x|^2}{(n^{-1} + 2|(-A)^{-\beta} x|^2)^2} + \frac{\sum_{h=1}^n \alpha_h^{-\beta} x_h v_h}{n^{-1} + 2|(-A)^{-\beta} x|^2}. \quad (4.6)$$

Proposition 4.4. *For any $r > 0$ there exists the surface measure σ_r on $B_{\sqrt{r}}$ determined by ν .*

Proof. Since obviously

$$\lim_{n \rightarrow \infty} F_g^n = F_g \quad \text{in } L^2(H, \nu; H)$$

and M^* is a closed operator, to prove that $F_g \in D(M^*)$ it is enough to show that there is a constant $K > 0$ such that

$$\|M^*(F_g^n)\|_{L^2(H, \nu)} \leq K, \quad \forall n \in \mathbb{N}. \quad (4.7)$$

To prove (4.7) we shall proceed in two steps.

Step 1. We show that $|(-A)^{-\beta} x|^{-2} \in L^{k+1}(H, \nu)$ for all $k \in \mathbb{N}$.

Let $d \in \mathbb{N}$ (to be chosen later) and let P be the orthogonal projector on $\text{span}(e_1, \dots, e_d)$. Write

$$\frac{1}{|(-A)^{-\beta}x|^2} \leq \frac{1}{|(-A)^{-\beta}Px|^2} \leq \frac{\alpha_d^\beta}{|Px|^2}.$$

Then it is enough to show that

$$\frac{1}{|Px|^2} \in L^k(H, \nu)$$

We apply Itô's formula to $\varphi_\epsilon(X(t))$ where X is the solution to (1.3) and

$$\varphi_\epsilon(x) = \frac{1}{(\epsilon + |Px|^2)^k}, \quad x \in H,$$

so that

$$(D\varphi_\epsilon(x), u) = -\frac{2k\langle Px, Pu \rangle}{(\epsilon + |Px|^2)^{k+1}} \quad x, u \in H$$

and

$$D_x^2\varphi_\epsilon(x)(u, v) = -2k\frac{(Pu, Pv)}{(\epsilon + |Px|^2)^{k+1}} + 4k(k+1)\frac{(Px, Pu)(Px, Pv)}{(\epsilon + |Px|^2)^{k+2}} \quad x, u, v \in H.$$

Therefore

$$\text{Tr} [D_x^2\varphi_\epsilon(x)] = -\frac{2kd}{(\epsilon + |Px|^2)^{k+1}} + 4k(k+1)\frac{|Px|^2}{(\epsilon + |Px|^2)^{k+2}}$$

and we have

$$\begin{aligned} \varphi_\epsilon(X(t)) - \varphi_\epsilon(x) - 2k\frac{|(-A)^{1/2}PX(t)|^2}{(\epsilon + |PX(t)|^2)^{k+1}} dt &= -2k\frac{\langle Pp(X(t)), PX(t) \rangle}{(\epsilon + |PX(t)|^2)^{k+1}} dt \\ &\quad - \frac{kd}{(\epsilon + |PX(t)|^2)^{k+1}} dt + 2k(k+1)\frac{|P(X(t))|^2}{(\epsilon + |P(X(t))|^2)^{k+2}} dt + dG_t, \end{aligned} \quad (4.8)$$

where G_t is a martingale.

We assume now that X is a stationary process with law ν . We integrate between 0 and 1 the identity (4.8) and we take the expectation. We get

$$\begin{aligned} kd \int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) &= 2k \int_H \frac{|(-A)^{1/2}Px|^2}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) \\ -2k \int_H \frac{\langle Pp(x), Px \rangle}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) &+ 2k(k+1) \int_H \frac{|Px|^2}{(\epsilon + |Px|^2)^{k+2}} \nu(dx) \\ &=: I_1 + I_2 + I_3. \end{aligned} \quad (4.9)$$

Let us estimate I_1 . Since

$$|(-A)^{1/2}Px|^2 \leq \alpha_d |Px|^2 \leq \alpha_d(\epsilon + |Px|^2),$$

given $\rho > 0$ and using Hölder's and Young's inequalities, we see that there is a constant $C_{\rho,k,d} > 0$ such that

$$\begin{aligned} |I_1| &\leq 2k\alpha_d \int_H \frac{1}{(\epsilon + |Px|^2)^k} \nu(dx) \leq 2k\alpha_d \left(\int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) \right)^{k/(k+1)} \\ &\leq C_{\rho,k,d} + \rho \int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx). \end{aligned} \quad (4.10)$$

Concerning I_2 , noticing that

$$\langle Pp(x), Px \rangle \leq |Pp(x)| |Px| \leq |Pp(x)| (\epsilon + |Px|^2)^{1/2},$$

using Hölder's and Young's inequalities and recalling (1.4), we see that there is a constant $C_{\rho,k} > 0$ such that

$$\begin{aligned} |I_2| &\leq 2k \int_H \frac{|Pp(x)|}{(\epsilon + |Px|^2)^{k+1/2}} \nu(dx) \\ &\leq \left(\int_H |Pp(x)|^{2k+2} d\nu \right)^{\frac{1}{2k+2}} \left(\int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) \right)^{\frac{2k+1}{2k+2}} \\ &\leq C_{\rho,k} + \rho \int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) \end{aligned} \quad (4.11)$$

Finally, obviously

$$|I_3| \leq 2k(k+1) \int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx). \quad (4.12)$$

By (4.10), (4.11) and (4.12) it follows that

$$\begin{aligned} kd \int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) &\leq C_{\rho,k,d} + C_{\rho,k} \\ &\quad + (2k(k+1) + 2\rho) \int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx). \end{aligned}$$

Now we choose d and ρ such that

$$d > 2(k+1), \quad kd > 2k(k+1) + 2\rho$$

and conclude that there exists $M > 0$ such that

$$\int_H \frac{1}{(\epsilon + |Px|^2)^{k+1}} \nu(dx) \leq M.$$

The conclusion follows letting $\epsilon \rightarrow 0$.

Step 2. Conclusion.

Set $\| |(-A)^{-\beta} x|^{-2} \|_{L^k(H, \nu)} = D_k$ and write $M^*(F_g^n) = J_1^n + J_2^n$, where

$$J_1^n = -\frac{\text{Tr} [(-A)^{-2\beta} P_n]}{n^{-1} + 2|(-A)^{-\beta} x|^2} + \frac{4|(-A)^{-2\beta} x|^2}{|(n^{-1} + (-A)^{-\beta} x|^2)^2}, \quad J_2 = \frac{\sum_{h=1}^n \alpha_h^{-\beta} x_h v_h}{n^{-1} 2|(-A)^{-\beta} x|^2}.$$

Since

$$|(-A)^{-2\beta} x|^2 \leq \alpha_1^{-2\beta} |(-A)^{-\beta} x|^2,$$

we have

$$J_1^n \leq \frac{\text{Tr} [(-A)^{-2\beta}] + 2\alpha_1^{-2\beta}}{|(-A)^{-\beta} x|^2} \quad (4.13)$$

Therefore by Step 1,

$$\|J_1^n\|_{L^2(H, \nu)} \leq D_1 (\text{Tr} [(-A)^{-2\beta}] + 2\alpha_1^{-2\beta}). \quad (4.14)$$

Concerning J_2^n we first notice that, thanks to (3.6) and (1.4) there is $L > 0$ such that

$$\begin{aligned} \left\| \sum_{h=1}^{\infty} \alpha_h^{-\beta} x_h v_h \right\|_{L^4(H, \nu)} &\leq \sum_{h=1}^{\infty} \alpha_h^{-\beta} \|x_h\|_{L^8(H, \nu)} \|v_h\|_{L^8(H, \nu)} \\ &\leq c_{8/7} \sum_{h=1}^{\infty} \alpha_h^{-\beta} \|x\|_{L^8(H, \nu)} \leq L. \end{aligned}$$

Now by Hölder's inequality it follows that

$$\|J_2^n\|_{L^2(H, \nu)} \leq L \| |(-A)^{-1} x|^{-2} \|_{L^8(H, \nu)} \leq LD_8. \quad (4.15)$$

The conclusion follows. □

4.2 Surface measure of a half-space

Let $n = 1, 2, 3$ and $g(x) = \langle x, b \rangle$, with $b \in H$. Then for any $r \in \mathbb{R}$, $\{g = r\}$ is a half-space which we denote by $S_{b,r}$.

Proposition 4.5. *Assume that*

$$\sum_{j=1}^{\infty} \alpha_j^{-1} a_j < \infty. \quad (4.16)$$

Then there exists the surface measure σ_r on $S_{a,r}$ determined by ν .

Proof. We have $Mg = (-A)^{-\beta}b$ and so

$$F_g = \frac{(-A)^{-\beta}b}{|(-A)^{-\beta}b|^2}.$$

By (3.21) it follows that

$$M^*(F_g) = \frac{1}{|(-A)^{-\beta}b|^2} \sum_{j=1}^{\infty} \alpha_j^{-\beta} v_j(x) b_j.$$

Therefore

$$\|M^*(F_g)\|_{L^2(H,\nu)} \leq \frac{C_2}{|(-A)^{-1}b|^2} \sum_{j=1}^{\infty} \alpha_j^{-1} b_j.$$

The conclusion follows using a similar approximation argument as above. □

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